

Probabilistic Modeling of Service Life for Structures Subjected to Chlorides

by Evan C. Bentz

Most current service life models of reinforced concrete structures subjected to chloride loading produce only a single deterministic time to first corrosion. Some feel that probabilistic modeling of corrosion is difficult to understand, tediously slow to calculate, and of limited value due to strong dependency on the assumed input parameters. Using a linearized analysis method, verified by a modified Monte Carlo technique, this paper shows that answers of acceptable accuracy can be obtained with only a few seconds of calculation on an inexpensive computer. The method is explained and shown with examples. Using such a technique, it is felt that probabilistic analyses of time to first corrosion can be demystified and achieve regular usage.

Keywords: corrosion; reinforced concrete; service life.

INTRODUCTION

Billions of dollars are spent in North America each year on repairs to bridges and parking decks prematurely damaged due to corrosion of reinforcing steel. Often this corrosion is a result of chloride ingress to the reinforcement from deicing salts, groundwater, or seawater. Many different methods of preventing this damage exist, ranging from mechanical barriers such as membranes, chemical protection schemes such as corrosion inhibitors, and protection through improved concrete quality. To assist in comparing these options on an equal footing, a program called Life-365^{1,2} has been written to estimate when first corrosion will occur for such structures.

Life-365, like many other programs, is deterministic in its operation, meaning that it will produce only one predicted time to the initiation of corrosion for one set of input parameters. This single output contrasts with the well-known fact that concrete structures are quite variable in properties both throughout the structure and in terms of quality of construction and materials from one project to another. It would be useful if programs like Life-365 were able to predict a range of expected times to first corrosion rather than a single value to allow owners to better manage risk. This paper proposes a method of quickly determining the cumulative distribution function of when first corrosion is expected for a reinforced concrete structure. Thus, designers and owners will be able to base the selection of corrosion protection schemes on a chosen risk of corrosion. It is hoped that this method can be implemented into Life-365 soon so that additional confidence in the analyses made with the program can be obtained.

RESEARCH SIGNIFICANCE

Service life prediction models are becoming more widely used as cost-benefit analyses become required practice to select from alternative corrosion protection strategies. It is important to provide engineers with a way to measure the confidence they should place in the results of these service life predictions. The most rigorous way to do this is to provide

full probabilistic results that allow the engineers to do whatever they wish, with the results ranging from a simple visual check of plots to complex financial calculations.

PROBABILISTIC METHODS FOR SERVICE LIFE MODELING

There are a number of methods for determining a probabilistic estimate of the time to first corrosion of a structure. These can be divided into two main categories: implicit and explicit.

Implicit probabilistic methods for service life modeling directly integrate equations for probability density functions into the equations modeling chloride transport. These methods usually result in a set of equations that directly predict the probability of corrosion at a given time. These have the advantage of a direct solution, but the disadvantage in the fact that the mathematics can be difficult for complex constitutive relationships. As an example, the Life-365^{1,2} primary modeling equations directly include the effects of temperature, time-dependent changes in the diffusion rate, and relatively complex changes in the surface chloride levels. For the case of the surface chlorides alone, there is no simple equation for the changes with time for the case of membranes or sealers, making it quite difficult to model with an implicit probabilistic method. Examples of an implicit probabilistic method can be found in papers by Sagüés and Kranc.³

Explicit methods do not require direct modification of the governing equations. One explicit method is based on the well-known reliability methods such as the first-order reliability method (FORM) or second-order reliability method (SORM). These methods use a limit-state function (g-function) that defines the difference between capacity and demand. Implicitly, these methods assume that the capacity and demand are bivariate normally distributed. For a reinforced concrete corrosion problem, the capacity might be defined as the chloride threshold required to depassivate the steel and the demand might be defined as the chloride concentration at the steel at a given time. While the former of these may be reasonably assumed to be normally distributed, from lack of information alone, the latter is less clearly so. FORM/SORM methods are most appropriate for the determination of very small probabilities of failure—for example, against structural collapse—and the level of detail employed may not be necessary for corrosion analyses. They also have the disadvantage of appearing as a black box in the analysis. Examples of a

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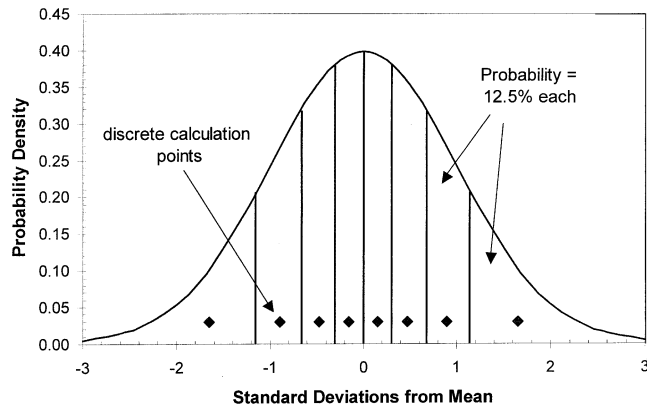


Fig. 1—Division of normal curve into discrete regions.

FORM-based probabilistic analysis of reinforced concrete service life can be found in papers by Lindvall,⁴ for example.

Another explicit method is the traditional Monte Carlo simulation. Using the assumed probability distributions of the input parameters, this method randomly selects inputs to the model and calculates the output. This is repeated until a sufficient histogram of output data is obtained and, as such, generally requires laborious calculation. Potential pitfalls are in the commonly used assumption that all the variables are independent, though that may not be the case. More importantly, it is not necessarily clear when sufficient iterations have been performed. An example of a method that uses Monte Carlo Simulation is a program from NIST by Ehlen.⁵

In this paper, an approximate explicit method that is based on the assumption of a probability distribution function and some simple calculations will be shown to work acceptably well. The method will be compared with a modified Monte Carlo method.

DETERMINISTIC MONTE CARLO METHOD

To calculate probabilities of failure for a structure, a modified Monte Carlo method will be used. Given the very limited information about the statistical variation of a number of the input parameters for service life analyses, it is inappropriate to calculate answers to a high level of precision. Accepting this coarse precision, it becomes possible to convert the Monte Carlo method into a deterministic one that has a predefined number of steps that will cover the entire input domain. Figure 1 shows a normal curve divided into eight sections, each with a probability of 0.125. At about the centroid of each region is a discrete point that represents the characteristic value for that region. The locations of these points are positive and negative 1.65, 0.89, 0.47, and 0.155 standard deviations from mean. By varying each set of the input parameters through all eight of these calculation points, it becomes possible to know exactly the number of iterations necessary to fully cover the input domain and produce a reasonable estimate of the solution. For example, if there are five input variables, each normally distributed, a total of $8^5 = 32,768$ iterations would be necessary to cover the input value domain. Thus, the method will produce a

probability distribution for the time to first corrosion of a structure without the question of deciding how many iterations are needed. Certainly 32,768 iterations is very significant, but this was chosen as the results will be used to confirm the quality of the simplified analysis method as follows. On an inexpensive 2002 computer, the Monte Carlo analysis took about 1 h to produce a set of results.

INPUT PARAMETERS FOR CHLORIDE MODEL

The input parameters for the modified Monte Carlo analysis explained in this paper will be based on Life-365.^{1,2} The program accepts basic mixture proportion information from the user and estimates the diffusion rate at 28 days as well as the rate of change of this diffusion rate with respect to time. The location and general exposure of the structure is used for temperature profile throughout the year as well as the surface chloride loading expected for the structure. As such, the equations that estimate the time to first repair are dependent on the following parameters:

1. Diffusion rate at 28 days, D_{ref} ;
2. Slope of diffusion plot with respect to time on log-log plot, m ;
3. Maximum surface chloride level (not constant, builds up over few years), C_s ;
4. Chloride threshold to initiate corrosion of steel, C_t ;
5. Clear cover to reinforcement, $cover$; and
6. Propagation period, t_p .

The statistical variation in each of these variables will be discussed as follows. In each case, the default values from Life-365 will be assumed to be the mean of the distribution, leaving the coefficient of variation to be estimated.

Variation in diffusion rate D_{ref}

The diffusion rate depends on many different parameters, but the model in Life-365 assumes that the water-cementitious material ratio (w/cm) is the prime variable. The top plot in Fig. 2 shows the results of 124 test results from an electrical migration test.⁶ While this is not the type of experiment used to generate the diffusion equation from Life-365, and hence the values it calculates are significantly different, it is useful for statistical purposes as there are so many data points from the same laboratory. The trend of the data with respect to w/cm is the same as predicted by Life-365. Also drawn on the top plot are bands 50% higher than the mean and 50% lower than the mean. The bottom plot shows the distribution of the ratio of the experimental diffusion rate to that predicted from the line through the center of the data. It can be seen that the data is reasonably normally distributed, with a coefficient of variation of 17%. Thus, it is possible, using a consistent test setup, consistent lab technicians, and single analysis method, to achieve reasonable stable measures of diffusion constant.

The top of Fig. 3 shows the data used to calibrate the equation for the diffusion constant used in the Life-365 development process.¹ These tests were bulk diffusion tests of a different type than in Fig. 2 and, more importantly, were performed in different labs by different operators. It can be seen that there is certainly more scatter to the data but that the general trend of a normally distributed result is acceptable. Not all the data shown in this figure were originally used in the calibration of the relationship in Life-365, and, as such, the average of the ratio of experimental to predicted diffusion rate is not equal to 1.0 but 1.15. Two outlier points are present in the data with measured diffusion rates 2.5 times the mean. Considering these points, the coefficient of variation of this

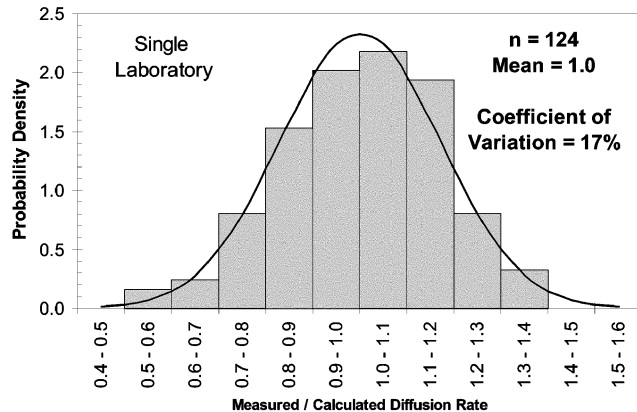
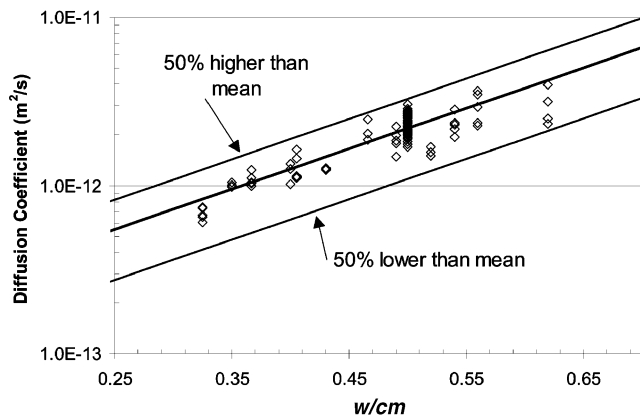


Fig. 2—Variation in diffusion rate within one lab.⁶

multi-lab study is 37%, and, ignoring them, the coefficient of variation is 21%, both significantly higher than that obtained from a single lab study.

For the analyses performed herein, it is not clear whether statistical parameters based on individual laboratory results or the results from a multi-lab study should be used. For the purposes of this paper, the coefficient of variation of the diffusion value will be assumed to be 25%.

Variation in diffusion rate with time, m

Unlike the diffusion rate, for which there is a clearly defined test procedure to determine its value, the other parameters in the model, including m , are a source of some contention. Work continues on determining standard tests for all these parameters, but for now a less scientific approach must be used. The parameter that controls the change in diffusion rate with time is called m , and in the program Life-365 it was determined based on a consensus approach, guided by what long-term data did exist. The parameter m varies between 0.2 and 0.6, with the higher end for concrete containing fly ash or slag and the lower end for regular portland cement concrete. For this paper, a normal distribution will be assumed with a fairly arbitrarily selected standard deviation of 0.05 or a coefficient of variation of 25% for ordinary portland cement concrete.

Variation in surface chloride level, C_s

Surface chloride levels vary dramatically from location to location within a structure, as well as between structures in the same city and from city to city across the continent. The variation from city to city is modeled by Life-365, but the

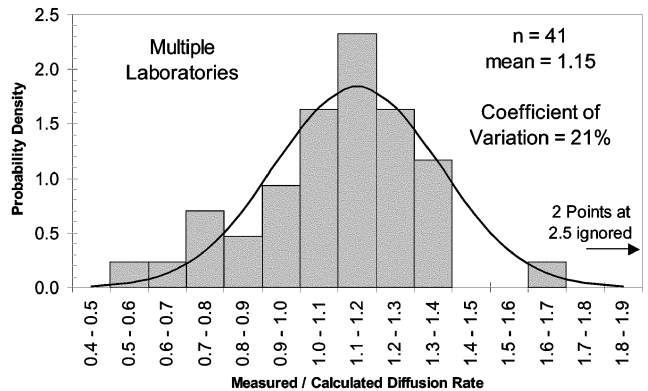
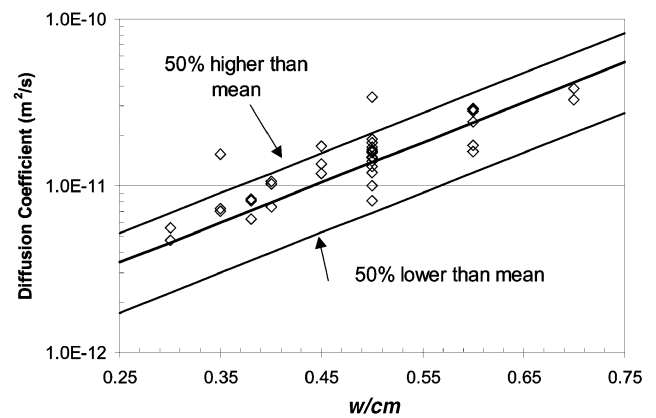


Fig. 3—Variation in diffusion rate between different labs.¹

other variations are not modeled. For this analysis, the coefficient of variation of surface chloride levels will be assumed to be 30%.

Variation in chloride threshold, C_t

The chloride threshold level to cause initiation of corrosion is also lacking a standard test method to quantify it. Glass and Buenfeld suggest that a threshold value can be expected to fall into a range of 0.03 to 0.07% total chloride by mass of concrete.⁷ Life-365 splits this range to provide the default value of 0.05%. Also based on this suggested range, the distribution in C_t will be assumed as normally distributed with a standard deviation of 0.01% total chloride by mass of concrete or a coefficient of variation of 20%.

Variation in cover

Cover depths can vary significantly over a structure due to quality of construction, reinforcement geometry, and, occasionally, quality of design. Assuming a tolerance of reinforcement placement of ± 10 mm, the distribution on cover depth will be taken as normally distributed with a standard deviation of 5 mm.

Variation in propagation period, t_p

Perhaps the least understood parameter in the life cycle model is the propagation period between the initiation of corrosion and the first major damage that necessitates repair. In recent ACI committee meetings, suggestions have been made that this parameter may exceed the value of 6 years used by Life-365 by almost an order of magnitude for high-performance concrete. Due to this severe uncertainty in evaluation, this parameter will be removed from the probabilistic

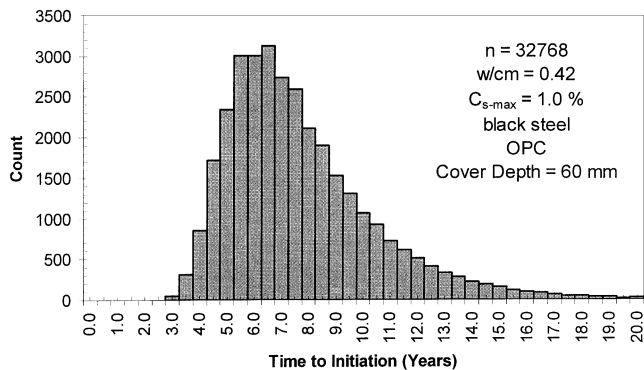


Fig. 4—Probability density function for time to first corrosion: base case analysis.

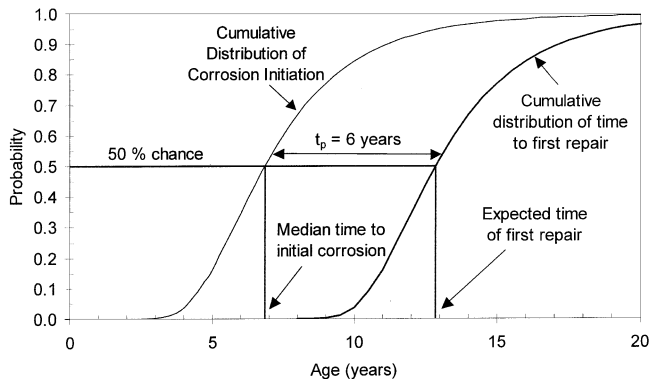


Fig. 5—Cumulative distribution of base case time to first corrosion.

analysis. This paper will therefore only refer to probabilistic times expected until first corrosion initiation, ignoring the propagation period. Much more research is needed to answer the question of an appropriate propagation period for reinforced concrete structures.

APPLICATION OF MONTE CARLO METHOD

The aforementioned method is applied to a simple analysis of a concrete structure. Table 1 summarizes the input parameters for this analysis. The surface chloride level is selected by Life-365 as being reasonable, if a little severe, for the Toronto region, with surface chlorides increasing linearly from zero to a maximum total chloride level of 1.0% of concrete mass achieved after 3.8 years. After this time, the chloride level is taken as constant. Any variation in this surface level is assumed to scale the entire profile, leaving the 3.8-year transition point intact. The temperature is adjusted by the time of year based on published temperature profiles for the city of Toronto. The concrete is assumed to have a water-cement ratio (w/c) of 0.42 with no fly ash, silica fume, or slag, producing the diffusion rate and m parameter listed in Table 1.

Figure 4 shows the results of applying the modified Monte Carlo method to these input parameters as a histogram of time to corrosion initiation. It can be seen that the resulting distribution appears more as a log-normal distribution than a normal one. This is not unexpected, as any arbitrarily distributed variables that are multiplied together will tend to produce a log-normal distribution by the central limit theorem. While the equations of Life-365 are not simple products internally, they are closer to that than sums. The median value of the distribution is 6.9 years, which is similar,

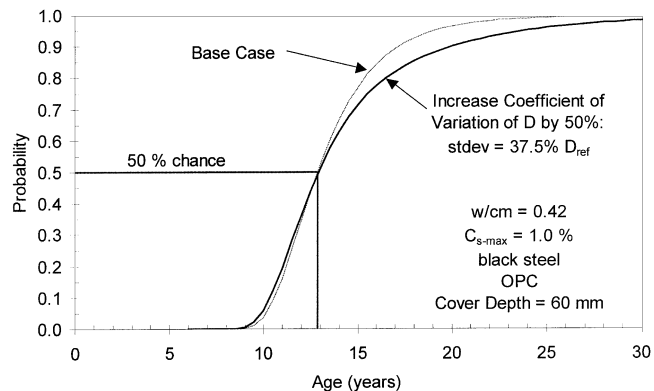


Fig. 6—Sensitivity of results to assumed variability of diffusion rate.

Table 1—Parameters for base case analysis

Variable	Units	Base value	Standard deviation	Coefficient of variation
D_{ref}	m^2/s	8.87E-12	2.22E-12	0.25
m	—	0.20	0.05	0.25
Max C_s	% Cl	1.0	0.30	0.30
C_t	% Cl	0.05	0.01	0.20
Cover	mm	60	5	0.08
t_p	years	6	—	—

though not identical, to the predicted time to corrosion initiation of 6.58 years if mean values are used for all parameters in Life-365. This difference is due to the nonlinear nature of the calculations for time to first corrosion.

Figure 5 shows the same results but converted to a cumulative distribution. Additionally, the 6-year propagation period is added to give estimates of when repairs will be needed for this structure. It can be seen that for this structure, the 10th percentile of corrosion occurring is about 2 years before the 50th percentile. That is, there is a 10% chance that the first repairs will be required after about 11 years rather than 13 years after construction. On the other hand, there is only a 10% chance the structure will survive 15 years until first repairs are required.

To explore the sensitivity of the results to the input parameters, additional analyses were performed with the aforementioned coefficients of variation of the input parameters increased by 50%, each in turn. Figure 6 shows the effect on the cumulative distribution if the coefficient of variation of the diffusion constant is increased from 25 to 37.5%. It can be seen that the distribution changes but not by an unreasonable amount. The cumulative profiles were found to be most sensitive to the m parameter, diffusion rate, and cover, with significantly less sensitivity to variability of the threshold value or variation in surface chloride levels.

An additional analysis was done for similar conditions as the previous base case example but with 40% of the cement replaced with class F fly ash. Life-365 suggests that this will result in identical parameters as the previous example but with the m parameter increased to 0.52. An analysis with Life-365 suggested that the expected time to first corrosion would be 31.1 years. Figure 7 shows the cumulative distribution results of the modified Monte Carlo analysis for this case. For this analysis, the standard deviation of the m parameter was kept at 0.05, essentially lowering the coefficient of variation to 9.6%. Note the 50th percentile of the Monte Carlo

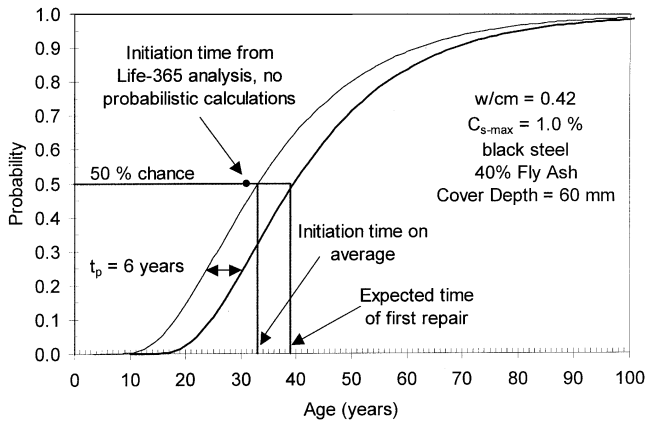


Fig. 7—Cumulative distribution of time to first corrosion for 40% fly ash.

initiation results, at 33.0 years, is significantly higher than the single analysis with mean parameters selected. This additional 1.9 years of life at the 50th percentile level is not accounted for in current models and is due to the nonlinear nature of the basic equations in the analysis.

SIMPLIFIED PROBABILISTIC ANALYSIS

While the shown results are very useful for the purposes of selecting alternative concrete protection strategies, the analysis time of 1 h per scenario means that it is not practical to use the Monte Carlo analysis method today. A simplified method will hereby be proposed that produces very similar answers in a matter of seconds of computation time.

Consider the analysis method inside a service life model, such as Life-365, as solving the following equation

$$t_i = f(D, m, C_s, C_t, Cover) \quad (1)$$

where t_i is the time to corrosion initiation. To estimate the coefficient of variation of this equation, linearize the equation about its mean values. Thus

$$t_i \approx \bar{t}_i + \left. \frac{\partial t_i}{\partial D} \right|_m (D - \bar{D}) + \left. \frac{\partial t_i}{\partial m} \right|_m (m - \bar{m}) + \left. \frac{\partial t_i}{\partial C_s} \right|_m \quad (2)$$

$$(C_s - \bar{C}_s) + \left. \frac{\partial t_i}{\partial C_t} \right|_m (C_t - \bar{C}_t) + \left. \frac{\partial t_i}{\partial Cover} \right|_m (Cover - \bar{Cover})$$

where the small m s indicate the partial derivative is evaluated at mean values. That is, the initiation time can be approximated as equal to the mean value of the initiation time plus the partial derivatives of Eq. (1) multiplied by differences from the mean. Because Eq. (2) is a linear combination of assumed independent random variables, its variance is

$$\sigma_{t_i}^2 = \left(\left. \frac{\partial t_i}{\partial D} \right|_m \right)^2 (\sigma_D)^2 + \left(\left. \frac{\partial t_i}{\partial m} \right|_m \right)^2 (\sigma_m)^2 + \left(\left. \frac{\partial t_i}{\partial C_s} \right|_m \right)^2 \quad (3)$$

$$(\sigma_{C_s})^2 + \left(\left. \frac{\partial t_i}{\partial C_t} \right|_m \right)^2 (\sigma_{C_t})^2 + \left(\left. \frac{\partial t_i}{\partial Cover} \right|_m \right)^2 (\sigma_{Cover})^2$$

This equation provides a simple method of estimating the variance and hence the standard deviation of the results of a

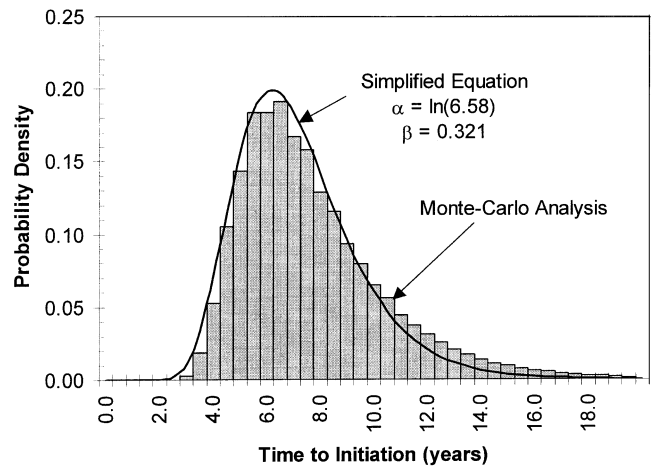


Fig. 8—Simplified analysis compared with Monte Carlo results for base case.

probabilistic analysis of time to corrosion initiation. If it is further assumed that the distribution, as suggested by Fig. 4, is log-normally distributed, and that the 50th percentile of the final analysis is equal to the results of the analysis performed with mean values, it is only necessary to determine the partial derivatives in Eq. (3) to approximate the entire distribution. These derivatives may be determined numerically, using any deterministic service life model. Thus, the estimation of the probabilistic results for an analysis will take six deterministic calculations, rather than the single calculation it takes otherwise: one analysis at the mean of the input parameters, plus one analysis to calculate each of the partial derivatives in Eq. (2) and (3).

The equation of the log-normal distribution for any time t greater than zero is

$$f(t) = \frac{1}{\sqrt{2\pi}\beta} t^{-1} e^{-(\ln t - \alpha)^2 / 2\beta^2} \quad (4)$$

where $\ln t$ is the natural logarithm of t , and α and β are statistical parameters calculated within the following three steps to perform the simplified analysis:

1) Assume that the 50th percentile of final distribution is the same as the result of the analysis performed with all parameters set to mean values. Thus, a regular analysis from programs as they stand today producing a given t_i allows the calculation of $\alpha = \ln t_i$;

2) Estimate partial derivatives of the five variables in the analysis by incrementing the value of these parameters by 10%, performing a deterministic analysis and dividing the change in time to corrosion by the increment. This must be performed for each variable in turn. Refer to Table 2; and

3) Calculate the β parameter from Eq. (3) and as shown in Table 2.

Figure 8 compares the results of the Monte Carlo analysis to the new estimation technique described previously and in Table 2. It can be seen that the results are excellent for this case. Figure 9 compares the cumulative distribution from the Monte Carlo analysis with this simplified method. Again, the fit is excellent up to a probability of corrosion of about 70 to 80%. As the most likely use of this probabilistic information is on the lower end of the distribution, it is judged acceptable that the upper region of the curve is modeled less well. Note

Table 2—Estimation of beta parameter*

Derivative calculation							
Variable	Units	Base value	Increment to variables	Modified variable	Initiation time, years	$\ln(t_i)$	Partial derivative
D_{ref}	m^2/s	8.87E-12	8.87E-13	9.76E-12	6.14	1.81	-7.803E+10
m	—	0.20	0.02	0.22	7.15	1.97	4.154
Max C_s	% Cl	1.0	0.1	1.1	6.38	1.85	-0.309
C_t	% Cl	0.05	0.005	0.055	6.82	1.92	7.165
Cover	mm	60	6	66	7.66	2.04	0.025
Beta parameter estimation							
Variable	Units	Partial derivative	Coefficient of variation	Standard deviation	Eq. (3) calculate	Contribution to variance, %	
D_{ref}	m^2/s	-7.803E+10	0.25	2.218E-12	0.030	29.1	
m	—	4.154	0.25	0.05	0.043	42.0	
Max C_s	% Cl	-0.309	0.3	0.3	0.009	8.3	
C_t	% Cl	7.165	0.2	0.01	0.005	5.0	
Cover	mm	0.025	0.083	5	0.016	15.6	
					Sum to get β^2	0.103	
					$\beta =$	0.321	

* $t_i = 6.58$ years; $\ln(t_i) = 1.884$.

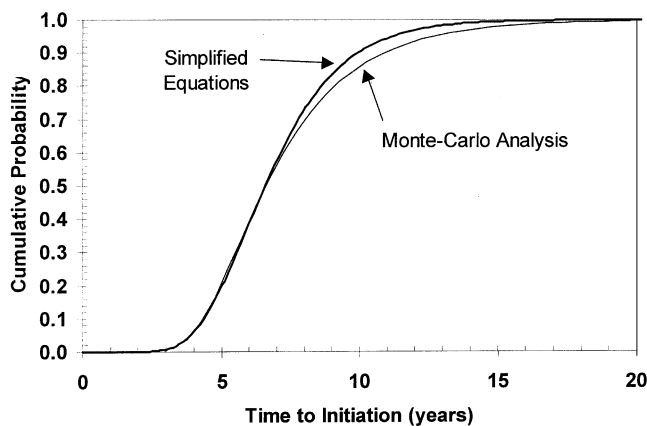


Fig. 9—Cumulative probability of corrosion initiation: simplified versus Monte Carlo.

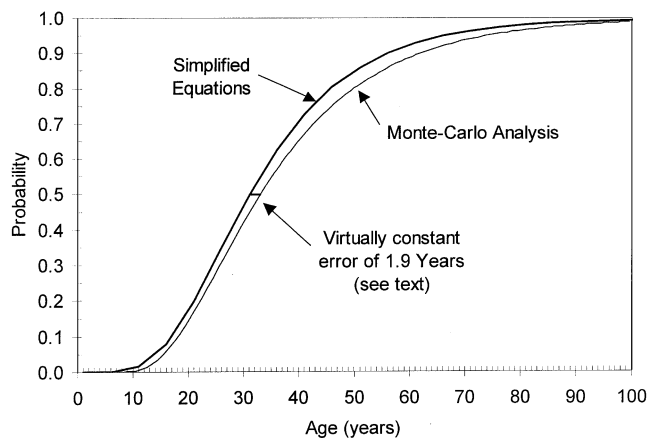


Fig. 10—Quality of simplified method prediction for 40% fly ash.

that the analysis results in the simplified method are obtained essentially instantaneously versus the hour taken for the comparison Monte Carlo analysis.

An additional advantage of the method is that it provides an indication of the source of the variation in the results. The final column on the bottom in Table 2 shows the contribution to variance, which indicates how much of the β^2 parameter

derives from each of the variables. In this example, it is confirmed that the results are most sensitive to the m and D parameters.

A weakness of the method is the assumption that the 50th percentile of the distribution is equal to the analysis result using mean values for all the variables. Figure 10 compares the results of the simplified method with the cumulative distribution for the 40% fly ash case mentioned previously. It can be seen that while the shape of the distribution is well modeled, there is an offset between the approximate method and the more complex Monte Carlo method of 1.9 years. As current analyses that do not use probabilistic calculations are judged to be acceptable even though they do not account for the short advantage shown by the Monte Carlo method, the results are considered acceptable.

COMMENTS ON USE OF PROBABILISTIC DATA

Using the approximate aforementioned method, there is no strong reason that probabilistic methods cannot be practically used for service life models. The method can be performed with the deterministic versions of programs, but it is hoped that future versions of Life-365 will be able to implement the method automatically.

The availability of probabilistic results merits some brief discussion of their use. Engineers are all familiar with the advantages of being conservative in design to ensure that there is no injury or loss of life caused by a failure within a structure. It is not unusual to design structural components with a probability of failure of one part in one million. This level of conservativeness could theoretically also be prescribed for service life models as well. The author feels that this would be a mistake, however, as the consequences of a service life failure are much less severe than a strength failure. Unlike structural design where a failure is never to occur, corrosion initiation is essentially guaranteed to eventually occur. As such, an early corrosion initiation will simply cause the structure to require repairs earlier than expected. While this will cause an unexpected expenditure of money, it will not cause loss of life.

Figure 11 shows a comparison of time to first corrosion for the base case analysis compared with the 40% fly ash case. It can be seen that the expected life extension from the use of fly ash in the concrete depends on the probability selected by

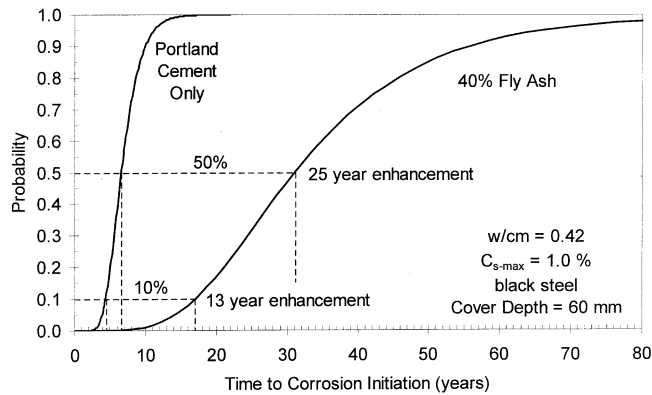


Fig. 11—Comparison of base case to 40% fly ash case time to first corrosion.

the designer. If an owner has many structures, such as a Department of Transportation, it is felt to be most appropriate to make service life decisions based on a 50th percentile of failure analysis. If some structures require repairs before this time, others can be expected to require repairs at a later age to balance out the overall costs. An owner with only a single structure may decide to aim to a lower probability of failure, say 10 rather than 50% to estimate the timing of future costs. In either case, the provision of an envelope of expected results provides much additional information allowing more intelligent decisions about future costs to be made.

CONCLUSION

Probabilistic analyses of service life of reinforced concrete structures can take many forms. A modified Monte Carlo analysis method is described that allows calculation of probability distributions within a known time of when the initiation of reinforcement corrosion is expected for a structure. Representative values of variability of the parameters used in analysis are provided. Analyses show that the resulting probability distribution is most sensitive to the diffusion rate

and the parameter that controls how that rate changes with respect to time. A simplified analysis method is also proposed that allows very fast estimation of the probability distributions of time to first corrosion in only a few seconds using any deterministic service life analysis program. The method is demonstrated with an example and it is felt that the method in this paper makes the calculation of probabilistic distributions of time to first corrosion very simple. Hence, it is now very practical to use such distributions in making engineering judgments of the selection of reinforcement corrosion protection strategies.

ACKNOWLEDGMENTS

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